

ANALYSIS OF VALUE AT RISK (VAR) ON STOCK PORTFOLIOS USING THE T-STUDENT COPULA METHOD

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Abstract:

This study aims to estimate the portfolio risk of BBRI and BBKA stocks using the t-Student Copula method. The method integrates the GARCH(1,1) model for marginal volatility and the t-Student Copula for the dependency structure through Monte Carlo simulation with 10.000 iterations. The results indicate a significant positive correlation, with a Kendall's Tau of 0,3242 and a copula correlation parameter of 0,4886. The degree of freedom of 12,9364 confirms the presence of tail dependence. The portfolio Value at Risk (VaR) is 1,9774% at the 95% confidence level and 3,1301% at the 99% level, with Expected Shortfall of 2,7571% and 4,0775%, respectively. In conclusion, the t-Student Copula provides more accurate and conservative risk estimates as it is capable of anticipating simultaneous extreme loss potential.

1. Introduction

Value at Risk (VaR) is a standard method for estimating the maximum potential investment loss at a specified confidence level (Christova et al., 2022). Although conventional Monte Carlo simulations are frequently utilized, this method is prone to producing underestimated VaR values because it assumes normal distribution and linear relationships. In reality, stock return data often exhibit non-linear patterns and heavy-tailed distributions. As a solution, the t-Student Copula method is applied due to its superior capability to capture non-linear relationships and extreme dependence during market crises more accurately (Carvalho & Sáfadi, 2022). This study applies this approach to a stock portfolio comprising PT Bank Central Asia Tbk. (BBKA) and PT Bank Rakyat Indonesia Tbk. (BBRI), two stocks that dominate the Indonesian capital market with the highest market capitalization and liquidity (Mutiar Fariandi & Asep Risman, 2025).

2. Literature Review

2.1 Time Series Analysis

Time series analysis is an observational method applied to variables collected sequentially in chronological order at consistent time intervals (Nabilah, 2017). This method is utilized to identify patterns, trends, and shifts in historical data in order to project or forecast future values (Aswi & Sukarna, 2006).

a. Stationarity

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A time series dataset is considered stationary if its fluctuations revolve around a constant mean and variance over time (Aswi & Sukarna, 2006). Statistical testing for stationarity can be performed using the Augmented Dickey-Fuller (ADF) test, where a dataset is declared stationary if the resulting *p-value* is less than the significance level (α).

Because stock price data are generally non-stationary, transformation into log returns is required to achieve stationarity using the following equation (Freiberger, 2008):

$$R_t = l_n \frac{p_t}{p_{t-1}}, \quad (1)$$

where:

R_t	= stock price return at time
p_t	= stock price at the current period
p_{t-1}	= stock price at the previous period
l_n	= natural logarithm

b. Autocorrelation Function (ACF)

The Autocorrelation Function (ACF) serves to describe the relationship between an observation and its previous values across various lags (Smith et al., 2024). The ACF can be formulated as follows:

$$\hat{\rho}_k = \frac{\hat{\gamma}_k}{\hat{\gamma}_0} = \frac{\sum_{t=1}^{n-k} (r_t - \bar{r})(r_{t+k} - \bar{r})}{\sum_{t=1}^n (r_t - \bar{r})^2}, \quad (2)$$

where:

$\hat{\rho}_k$	: estimated autocorrelation at lag k
$\hat{\gamma}_k$	: estimated autocovariance at lag k
$\hat{\gamma}_0$	: estimated variance (autocovariance at lag 0)
r_t	: stock return data at time t
r_{t+k}	: stock return data at time $(t - k)$
$\bar{r}$	: mean of stock return data
n	: total number of observations

c. Partial Autocorrelation Function (PACF)

The Partial Autocorrelation Function (PACF) measures the correlation between two observations at different lags after removing the influence of intermediate lags (Zhao et al., 2023).

The PACF can be mathematically expressed as follows (Zhao et al., 2023):

$$\hat{\phi}_{k+1,k+1} = \frac{\hat{\rho}_{k+1} - \sum_{j=1}^k \hat{\phi}_{k,j} \hat{\rho}_{k+1-j}}{1 - \sum_{j=1}^k \hat{\phi}_{k,j} \hat{\rho}_j}, \quad (3)$$

where:

$\hat{\phi}_{1,1} = \hat{\rho}_1, \hat{\phi}_{k+1,j} = \hat{\phi}_{k,j} - \hat{\phi}_{k+1,k+1} \hat{\phi}_{k,k+1-j},$	$j = 1, 2, \dots, k$
$\hat{\phi}_{k+1,k+1}$	: estimated partial autocorrelation at lag k
$\hat{\phi}_{k,j}$	: estimated partial autocorrelation at lag $k - 1$ and position j
$\hat{\rho}_{k+1}$	: estimated autocorrelation at lag- $(k + 1)$
$\hat{\rho}_{k+1-j}$	: estimated autocorrelation at lag- $(k + 1 - j)$
$\hat{\rho}_j$	: estimated autocorrelation at lag j .

d. Akaike Information Criterion (AIC)

The Akaike Information Criterion (AIC) is used to objectively identify the best statistical model among several alternative models based on its minimum value (*Minimum AIC Estimate*) (Akaike, 1974). This procedure applies the principle of parsimony by penalizing the addition of unnecessary parameters to prevent overfitting. Mathematically, AIC is formulated as follows:

$$AIC = -2 \ln(L) + 2k, \quad (4)$$

where:

$\ln(L)$ = maximum log-likelihood value of the model being evaluated (measuring model fit to the data)

k = number of independently estimated parameters in the model (serving as the penalty component)

2.2 ARIMA (Model Identification)

The Autoregressive Integrated Moving Average (ARIMA) model is a univariate forecasting method that relies on past values and random fluctuations of the variable itself (Khoiri, 2022). Determining the order of the Autoregressive (p) AND Moving Average (q) components is visually conducted by observing the patterns of the Autocorrelation Function (ACF) and Partial Autocorrelation Function (PACF) plots (Aswi & Sukarna, 2006). The characteristics of both plots are summarized in Table 1.

Table 1. ACF and PACF Patterns for Model Identification

ARIMA Model	ACF	PACF
Autoregressive AR (p)	Dies down	Cut off after lag p
Moving Average MA (q)	Cut off after lag q	Dies down
Autoregressive Moving Average ARMA (p, q)	Dies down	Dies down

(Aswi & Sukarna, 2006)

2.3 Autocorrelation Test (White Noise Assumption Test)

Evaluating the white noise assumption of model residuals is visually conducted through the Autocorrelation Function (ACF) plot. A model is considered to satisfy this assumption if no lags on the ACF plot exceed the boundaries of the 95% confidence level. The hypothesis testing is structured as follows:

H_0 : Residuals exhibit white noise properties (no autocorrelation).

H_1 : Residuals do not exhibit white noise properties (autocorrelation is present).

Formally, statistical testing using the Ljung-Box test is formulated as follows (Box & Pierce, 1970):

$$Q = n(n+2) \sum_{k=1}^k \frac{\rho_k^2}{n-k}, \quad (5)$$

where:

n : total number of observations

k : maximum number of tested lags

ρ_k : sample autocorrelation of the residuals at lag k

The decision rule for the Ljung-Box test dictates that H_0 is rejected if $p\text{-value} < \alpha$, indicating significant autocorrelation across lags.

2.4 Heteroskedasticity Test

Heteroskedasticity testing in stock return data is conducted to detect the phenomenon of volatility clustering. The detection of Autoregressive Conditional Heteroscedasticity (ARCH) effects in model residuals is performed using the ARCH-Lagrange Multiplier (ARCH-LM) test (Fadhilah et al., 2024). The hypothesis testing is formulated as H_0 (no ARCH effect present) versus H_1 (ARCH effect present) using the following test statistic:

$$LM = nR^2, \quad (6)$$

where:

n : total number of observations (sample size)

R^2 : coefficient of determination from the regression of squared residuals on their lagged squared values

The null hypothesis H_0 is rejected if $LM > X^2_{(\alpha,m)}$ or if $p - value < \alpha = 0,05$, and m representing the degrees of freedom corresponding to the chosen ARCH model order.

2.5 Kendall's Tau (τ) Rank Correlation

Kendall's tau rank correlation is a non-parametric measure of concordance used to evaluate the strength of the dependency relationship between two random variables. This method is superior to Pearson's linear correlation because it does not require the assumption of a normal distribution and is capable of capturing non-linear relationships in stock returns by separating the dependency structure from the marginal distributions (Pratama et al., 2025).

a. Concept of Concordance

The definition of Kendall's tau is based on the concepts of concordant and discordant data pairs. Suppose there are two independent observation pairs, (X_1, Y_1) and (X_2, Y_2) drawn from the same joint distribution (Manstavicius, 2024). The pair is said to be concordant if $(X_1 - X_2)(Y_1 - Y_2) > 0$ and discordant if $(X_1 - X_2)(Y_1 - Y_2) < 0$. Mathematically, Kendall's tau is defined as the difference between the probability of concordance and the probability of discordance:

$$\tau = P[(X_1 - X_2)(Y_1 - Y_2) > 0] - P[(X_1 - X_2)(Y_1 - Y_2) < 0], \quad (7)$$

where:

τ : Kendall's tau rank correlation coefficient.

P : probability of an event.

X_1, X_2 : two independent observations of the first variable (e.g., BBRI stock returns).

Y_1, Y_2 : two independent observations of the second variable (e.g., BBKA stock returns).

$(X_1 - X_2)(Y_1 - Y_2) > 0$: condition indicating that the data pair is concordant, meaning both variables move in the same direction.

$(X_1 - X_2)(Y_1 - Y_2) < 0$: condition indicating that the data pair is discordant, meaning both variables move in opposite directions.

The value of τ ranges between $-1 \leq \tau \leq 1$. A value approaching 1 indicates a strong positive dependency, whereas a value approaching 0 indicates the absence of rank dependency between assets.

b. Application to the t-Student Copula

In this study, Kendall's tau is utilized as the primary instrument to estimate the correlation parameter of the t-Student Copula. Unlike Archimedean copulas (such as Gumbel or Clayton), the t-Student Copula belongs to the family of elliptical copulas, which are characterized by a correlation parameter ρ and degrees of freedom ν . There is a direct functional relationship between Kendall's tau (τ) and the correlation parameter ρ in a t-Student Copula, expressed as:

$$\rho = \sin\left(\frac{\pi}{2}\tau\right), \quad (8)$$

This approach provides parameter estimates that are consistent and robust against outliers in the return data of BBRI and BBKA during periods of extreme market volatility. Furthermore, Kendall's tau satisfies Scarsini's Axioms, ensuring that this measure serves as a mathematically consistent representation of dependency (Manstavicius, 2024).

2.6 Copulas

A copula is a function used to join multiple marginal distribution functions into a single multivariate cumulative distribution function, enabling the precise modeling of dependency structures between random variables. According to Sklar's Theorem (1959), the joint Cumulative Distribution Function (CDF) $H(x,y)$ of two random variables X and Y with continuous marginal distributions $F(x)$ and $G(y)$ is defined as (Nelsen, 2006):

$$H(x, y) = C(F(x), G(y)) = C(u, v), \quad (9)$$

where:

$F(x)$: marginal CDF of variable X

$G(y)$: marginal CDF of variable Y

$C(u, v)$: Copula function, defined on the unit square with $u = F(x)$ dan $v = G(y)$

Here, C represents the copula function that captures the statistical relationship between X and Y (Fadhilah et al., 2024).

The primary advantage of copulas in financial risk management lies in their ability to decouple the modeling of marginal distributions from their joint dependency structure. This offers the flexibility required to capture tail dependence specifically, the joint probability of co-occurring extreme events during financial crises (Hariscandra et al., 2016).

a. t-Student Copula

The t-Student Copula is widely utilized in financial risk management due to its capacity to model heavy-tailed distributions and symmetric tail dependence through its degrees-of-freedom parameter (ν) (Härdle et al., 2008). The choice of this method aligns with the leptokurtic characteristics of BBRI and BBKA stock returns, thereby offering a more realistic representation of co-occurring extreme market movements (contagion effect) for Value at Risk (VaR) estimation.

The t-Student Copula function is defined as:

$$C_t(u, v; \nu, \delta) = t_{\nu, \delta}(t_{\nu}^{-1}(u), t_{\nu}^{-1}(v)), \quad (10)$$

where:

$t_{\nu, \delta}$: joint CDF of the bivariate Student's t-distribution with ν degrees of freedom and linear correlation parameter δ

t_{ν}^{-1} : quantile function (inverse CDF) of the univariate Student's t-distribution with ν degrees of freedom

2.7 Value at Risk (VaR)

Value at Risk (VaR) is a statistical method used to estimate the maximum potential loss that a portfolio may incur over a specified time horizon at a given confidence level under normal market conditions (Jorion, 2007). In general, there are three main approaches to calculating VaR (Wipplinger, 2007):

a. Variance-Covariance (delta normal)

This approach assumes a normal distribution and calculates VaR based on the mean and volatility of returns:

$$VaR = Pz_{0,95}\sigma\sqrt{t}, \quad (11)$$

where:

P : initial portfolio investment value

$z_{0,95}$: initial portfolio investment value 95%

σ : volatility of single-asset returns

$\sqrt{t}$: time horizon

b. Monte Carlo Simulation

This method generates random return scenarios based on specified distributional assumptions:

$$VaR = \mu - (Z \times \sigma), \quad (12)$$

where:

VaR : maximum potential loss

μ : mean asset return

Z : quantile value associated with the confidence

σ : standard deviation of returns

c. Historical Simulation

This approach utilizes actual historical return data to estimate the empirical distribution of losses:

$$VaR_{(1-\alpha)} = \mu(R) - R\alpha, \quad (13)$$

where:

$VaR_{(1-\alpha)}$: maximum potential loss

$\mu(R)$: mean asset return

$R\alpha$: percentile value corresponding to the maximum loss at level α

Monte Carlo simulation is considered superior for VaR estimation due to its flexible capacity to capture non-linear price risks, volatility shifts, and extreme scenarios (Maruddani & Purbowati, 2012). The primary advantage of this method lies in its capability to generate the entire probability density function rather than a single quantile value, thereby enabling the evaluation of expected losses exceeding the VaR threshold.

2.8 Model Validity Evaluation (Backtesting)

Backtesting is conducted to verify the accuracy of the VaR model by comparing actual portfolio losses against historical VaR estimates (Saifullah et al., 2024). A violation occurs when actual losses exceed the estimated VaR. An excessively high number of violations indicates that the model underestimates risk, whereas too few violations suggest an overly conservative model.

Formal evaluation is performed using Kupiec's Proportion of Failure (POF) test based on the Likelihood Ratio (LR) (Saifullah et al., 2024):

$$LR_{POF} = -2 \ln \left[\frac{(1-\alpha)^{T-N} \alpha^N}{(1-\hat{p})^{T-N} \hat{p}^N} \right], \quad (14)$$

where:

T : total number of observations.

N : number of observed violations.

α : target violation probability (e.g., 0,05 for VaR 95% or 0,01 for VaR 99%).

$\hat{p}$: actual proportion of violations, calculated $\hat{p} = \frac{N}{T}$.

This test statistic follows a Chi-Square distribution X^2 with one degree of freedom ($df = 1$). A model is considered valid if $LR_{POF} < \chi^2_{(1,\alpha)}$ or if the p -value is greater than the significance level α . In accordance with the Basel Committee on Banking Supervision (BCBS) standards, a 99% confidence level is required to test the model's robustness in capturing tail events. A valid test result confirms that the correlation parameter (ρ) and degrees of freedom (ν) of the t-Student Copula effectively capture tail dependence.

2.9 Investment

Investment is the commitment of funds at present with the expectation of generating future financial returns (Karlina & Sulistianingsih, 2024). Among various financial instruments, stocks offer high potential returns that are commensurate with their inherent risk levels.

This study focuses its analysis on the shares of PT Bank Rakyat Indonesia Tbk. (BBRI) and PT Bank Central Asia Tbk. (BBCA)—two blue-chip stocks that dominate market capitalization and liquidity within the Indonesian banking sector (Komansilan et al., 2024). The selection of these two market leaders is based on three key considerations:

- Their performance closely reflects broader macroeconomic conditions.
- As assets within the same sector, evaluating their tail dependence structure is critical to anticipating simultaneous downside risk during banking crises.
- Their high liquidity produces robust and consistent time-series data suitable for complex econometric modeling.

2.10 Risk

Investment risk is reflected by the variability or volatility of an asset's historical returns. Based on its sources, risk is divided into two primary categories (Parendra et al., 2020):

- a. Macroeconomic risk that impacts the market as a whole (such as inflation and interest rate policy changes) and cannot be eliminated through diversification.
- b. Firm-specific or internal risk that can be minimized through portfolio construction (diversification).

3. Research Methodology

This study adopts a quantitative research design aimed at analyzing the portfolio risk of banking sector stocks using Value at Risk (VaR) estimation based on the t-Student Copula method. This approach is applied to model the dependency structure between assets while accounting for heavy-tailed properties and extreme market co-movements (tail dependence), which frequently occur in financial return data during periods of high volatility.

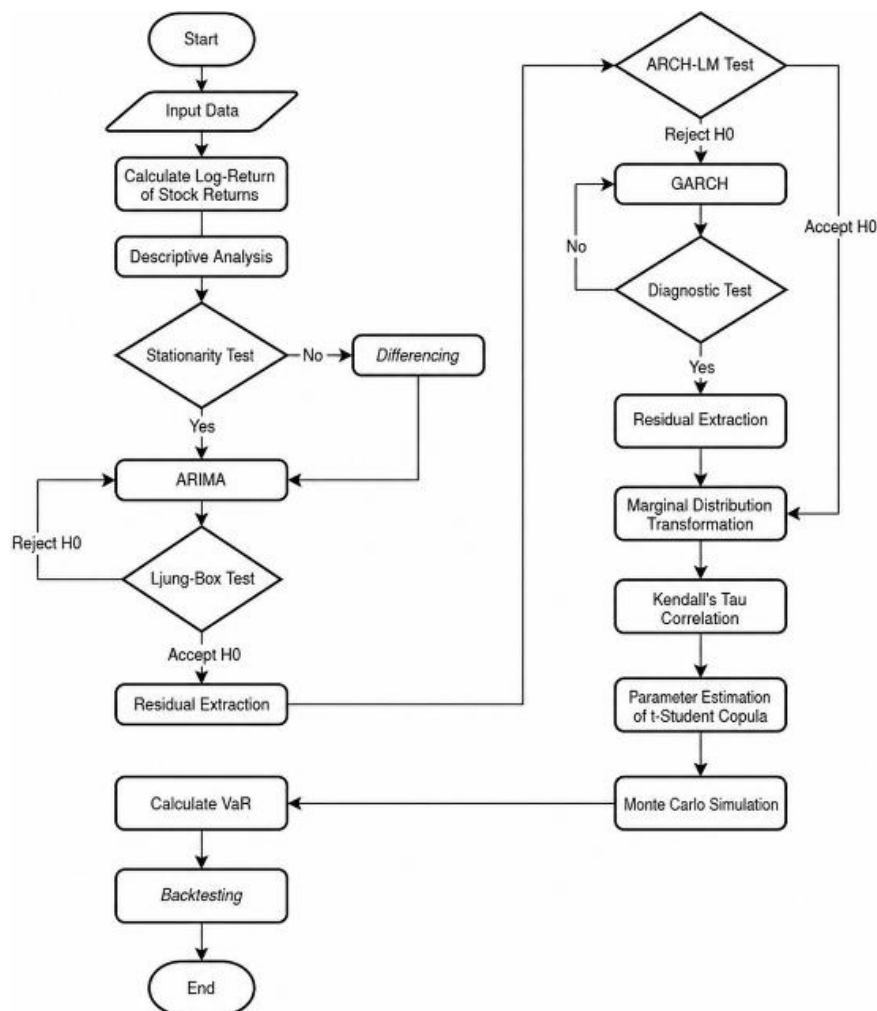


Figure 1. Data Analysis Technique Flowchart

4. Results and Discussion

4.1 Stock Price Data

This study uses 1,206 daily closing price observations for BBRI and BBKA shares, obtained from Yahoo Finance

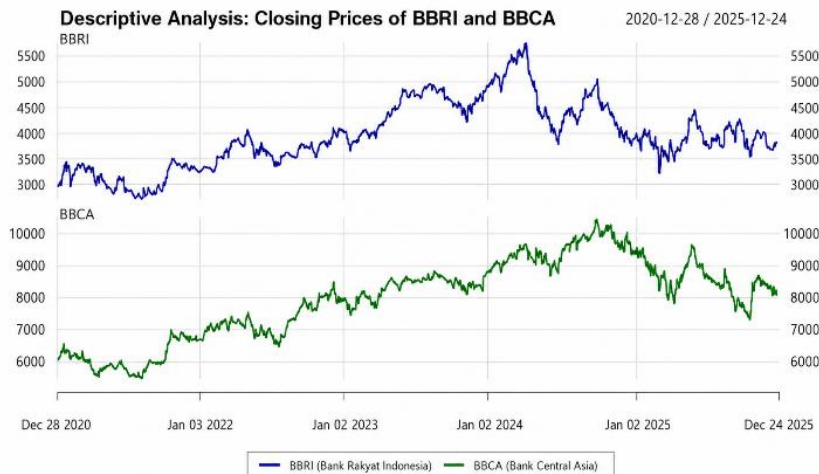


Figure 2. Stock Price Comparison

The raw stock price dynamics depicted in Figure 2 demonstrate a long-term upward trend (uptrend), reflecting the post-COVID-19 recovery of the banking sector. However, the mean and variance of the data change over time, indicating non-stationarity and rendering the raw prices unsuitable for direct inferential analysis.

In addition to non-stationarity, co-movement patterns between BBRI and BBKA are observed across specific periods, such as the growth phase (2021), market correction (mid-2022), and recovery phase (2023–2024). This tendency provides preliminary evidence of a positive correlation and dependency structure between the two assets.

4.2 Stock Price Log-Returns

Daily stock price log-returns were calculated using Equation (1). The log-return plots for BBRI and BBKA stock are presented in Figure 3.

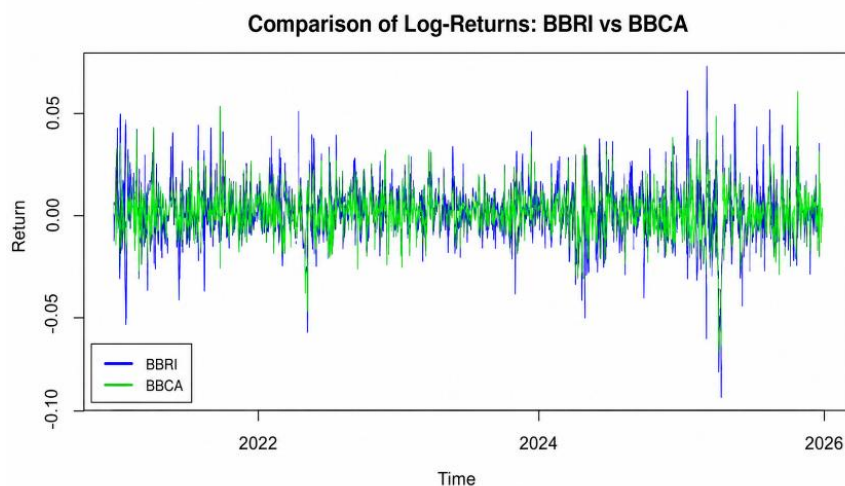


Figure 3. Comparison Plot of BBRI and BBKA Stock Log-Returns

The visualization of log-returns in Figure 3 indicates that the data series have achieved stationarity, as characterized by consistent oscillations around a constant zero mean. Furthermore, the volatility clustering phenomenon is observed, where fluctuations of large or small amplitudes tend to cluster together within the same time periods for both stocks.

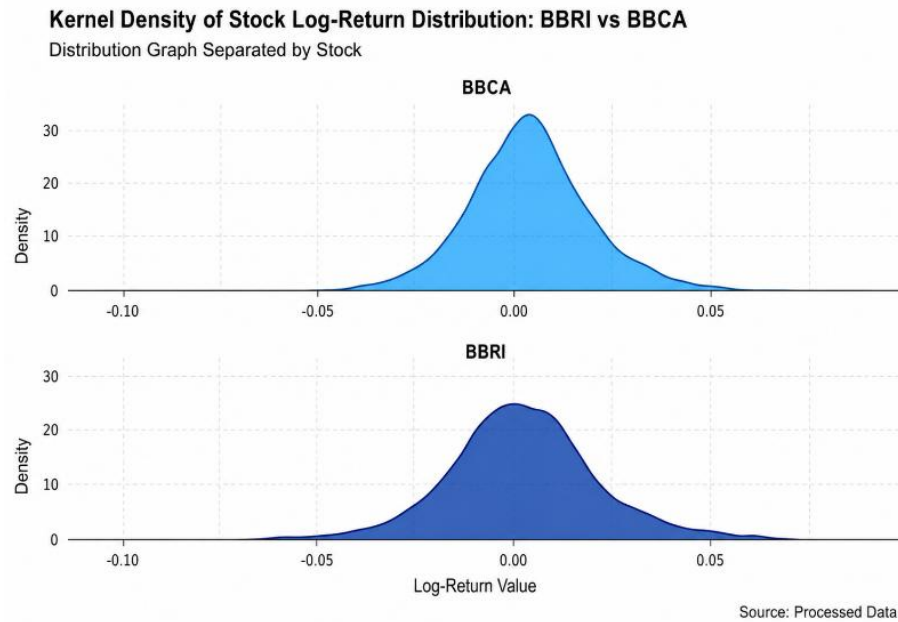


Figure 4. Bell Curve of BBRI and BBKA Stock Log-Return Distributions

Based on the bell curves in Figure 4, the log-return distributions of both assets exhibit higher peakedness and heavier (fatter) tails compared to a standard normal distribution. The presence of heavy tails on both the left and right sides indicates that the probability of extreme daily return occurrences for BBRI and BBKA is higher than what is assumed under normality.

The occurrence of the lower (left) tail is driven by negative market shocks or bearish sentiment that triggers mass selling, whereas the upper (right) tail is propelled by optimistic bullish sentiment. These characteristics confirm that the log-returns of BBRI and BBKA stock are leptokurtic a stylized fact commonly found in financial data, signifying that the probability of extreme returns (both heavy losses and large gains) is substantially higher than predicted by a standard normal distribution.

4.3 Descriptive Statistics

The statistical characteristics of daily returns for BBRI and BBKA over the observation period are presented in Table 1.

Table 1. Descriptive Statistics of BBRI and BBKA Stock Returns

Descriptive Statistic	BBRI	BBKA
Mean	0,000199	0,000244
Median	0,000000	0,000000
Standard Deviation	0,018202	0,014592
Minimum	-0,106733	-0,089153
Maximum	0,088251	0,073427
Skewness	0,050297	0,182793
Kurtosis	5,568257	5,602553

The positive daily mean returns indicate a long-term growth trend, with BBKA (0,000244) performing slightly higher than BBRI (0,000199). BBRI stock exhibits higher risk/volatility (standard deviation of 0,018202) and a wider range of price fluctuations compared to BBKA (0,014592.)

The positive skewness values for both stocks reflect right-skewed distributions. This departure from normality is further substantiated by high kurtosis estimates (BBRI: 5,568257; BBKA: 5,602553) both exceeding the normal threshold of 3 (leptokurtic). This heavy-tailed (fat-tailed) condition confirms the necessity of GARCH and Copula modeling to capture non-linear and heavy-tailed data characteristics.

4.4 Stationarity Test

Stationarity testing was performed using the Augmented Dickey-Fuller (ADF) test on the log-return series of each stock under the following hypotheses:

- H_0 : The data series is non-stationary (contains a unit root).
 H_1 : The data series is stationary (does not contain a unit root).

Table 2. Stationarity Test Results for BBRI and BBKA Stock Returns

Stock	ADF Test	<i>p-value</i>
BBRI	-10,945	0,01
BBKA	-10,874	0,01

Based on Table 2, the ADF test statistics for BBRI (-10,945) and BBKA (-10,874) yield a *p-value* of 0,01 ($< 0,05$). Consequently, the null hypothesis is rejected, confirming that the log-return series for both BBRI and BBKA are stationary at level.

4.5 ARIMA Modeling

Autoregressive Integrated Moving Average (ARIMA) modeling was conducted to capture linear dependency patterns in the log-return series of BBRI and BBKA stock. This model serves as the conditional mean equation, yielding residuals used to detect the presence of heteroskedasticity effects.

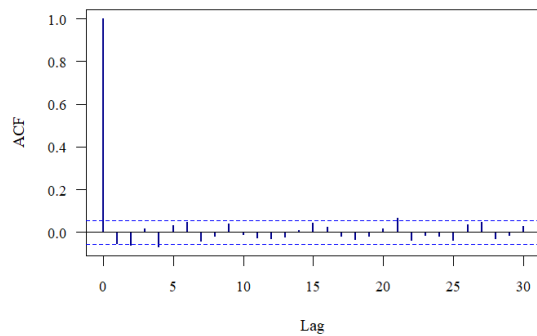


Figure 5. ACF Plot of BBRI Stock

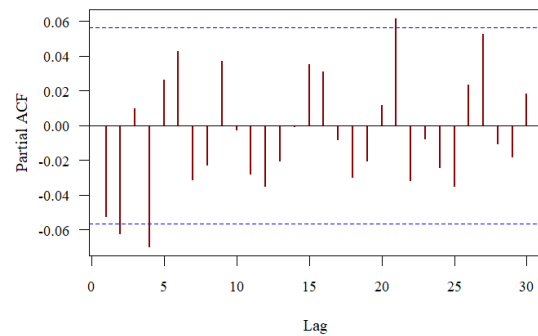


Figure 6. PACF Plot of BBRI Stock

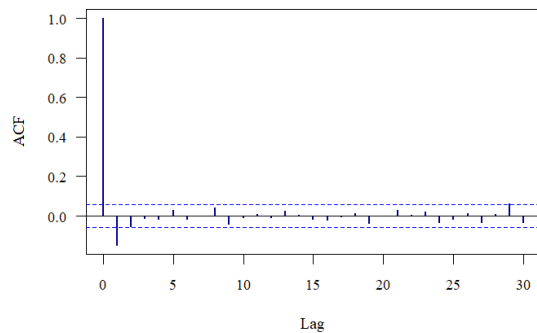


Figure 7. ACF Plot of BBKA Stock

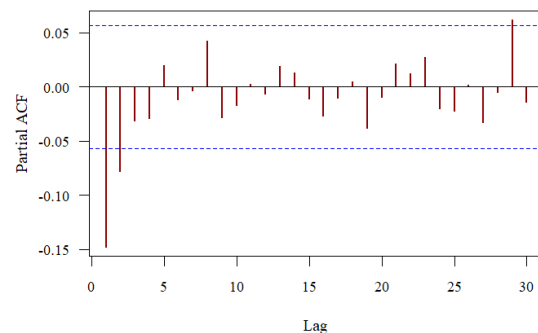


Figure 8. PACF Plot of BBKA Stock

a. Model Identification

Model identification was performed using Autocorrelation Function (ACF) and Partial Autocorrelation Function (PACF) plots. The ACF and PACF plots for BBRI stock display significant spikes in the PACF up to lag 4 while the ACF dies off, yielding 24 candidate models ranging from ARIMA(4,0,4) to ARIMA(0,0,1). For BBKA stock, the ACF and PACF patterns decay more rapidly, resulting in 5 candidate models: ARIMA(2,0,1), (1,0,1), (2,0,0), (1,0,0), and (0,0,1).

b. Model Estimation and Selection

The selection of the optimal model was based on the lowest Akaike Information Criterion (AIC) value among the candidate models evaluated, as summarized in Table 3.

Table 3. ARIMA Model Selection

Stock	ARIMA Model	AIC Value
BBRI	(3,0,2)	-6235,03
	(2,0,3)	-6235,20
	(1,0,0)	-6235,45
	(0,0,1)	-6235,91
	⋮	⋮
	(4,0,2)	-6242,56
	(2,0,4)	-6243,07
BBKA	(1,0,0)	-6791,21
	(0,0,1)	-6795,46
	(2,0,1)	-6795,96
	(2,0,0)	-6796,41
	(1,0,1)	-6797,85

Based on Table 3, the ARIMA(2,0,4) model was selected for BBRI stock (AIC: -6243,07) and ARIMA(1,0,1) model was chosen for BBKA stock (AIC: -6797,85).

c. Diagnostic Checking of ARIMA Models

The Ljung-Box test was conducted to ensure that the residuals exhibit white noise properties (H_0 : Residuals are white noise), while the ARCH-LM test was performed to detect heteroskedasticity (H_0 : No ARCH effect present). The results of both diagnostic tests are presented in Table 4.

Table 4. Diagnostic Test Results for ARIMA Model Residuals (Ljung-Box and ARCH-LM)

Stock	Ljung-Box p -value	ARCH-LM p -value
BBRI (2,0,4)	0,9998	< 0,0001
BBKA (1,0,1)	0,9042	< 0,0001

As shown in Table 4, the Ljung-Box test p -value for both stocks exceed the significance threshold $> 0,05$ (failing to reject H_0), confirming that the residuals behave purely as white noise. Conversely, the ARCH-LM test p -value are less than 0,05 (rejecting H_0), violating the homoskedasticity assumption and confirming the presence of ARCH/heteroskedasticity effects. This validates the necessity of proceeding with advanced volatility modeling using GARCH framework.

4.6 GARCH Modeling

Following the verification of ARCH effects in the ARIMA model residuals, volatility modeling was conducted using Generalized Autoregressive Conditional Heteroskedasticity (GARCH) with a Student's t-distribution assumption (std) to capture fat-tailed behavior.

a. Optimal GARCH Order Selection

The optimal GARCH order was determined by comparing Akaike Information Criterion (AIC) values across several order combinations (p, q), as presented in Table 5.

Table 5. GARCH Order Selection Results

Stock	GARCH(1,1)	GARCH(1,2)	GARCH(2,1)	Selected Order
BBRI	-5,2999	-5,2985	-5,2983	GARCH(1,1)
BBCA	-5,7477	-5,7468	-5,7462	GARCH(1,1)

Based on Table 5, the GARCH(1,1) model was selected as the optimal specification for both BBRI (AIC: -5,2999) and BBCA (AIC: -5,7477) yielding the lowest AIC values while satisfying the principle of parsimony.

b. Parameter Estimation GARCH(1,1)

Parameter estimation was performed to measure the impacts of past shocks (ARCH effect) and past volatility (GARCH effect) on current conditional variance.

Table 6. Optimal Parameter Estimates for BBRI and BBCA Stock

Parameter	BBRI Estimate	<i>p-value</i>	BBCA Estimate	<i>p-value</i>
μ (Mean)	0,000208	0,604878	0,000001	0,997156
ω (Omega)	0,000008	0,003554	0,000011	< 0,001
α_1 (Alpha)	0,062009	< 0,001	0,078945	< 0,001
β_1 (Beta)	0,913415	< 0,001	0,872417	< 0,001
Shape ν	6,428353	< 0,001	5,805325	< 0,001

BBCA Estimate α and β parameters are highly statistically significant ($p\text{-value} < 0,001$). The high magnitude of the β coefficients (BBRI: 0,9134; BBCA: 0,8724) indicates strong volatility persistence, meaning that market shocks require a relatively extended period to dissipate. The shape parameters (ν) ranging between 5,8 and 6,4 confirm heavy-tailed properties in the residual series.

c. Diagnostic Checking of GARCH Models

Diagnostic checks on the standardized residuals were conducted to ensure that the GARCH(1,1) specification successfully removed remaining autocorrelation and heteroskedasticity patterns.

Table 7. Diagnostic Test Results for GARCH(1,1) Standardized Residuals

Test Type	Test Name	BBRI <i>p-value</i>	BBCA <i>p-value</i>	Remark
Autocorrelation	Weighted Ljung-Box (Lag 5)	0,0772	1,0000	White Noise
Heteroskedasticity	Weighted ARCH-LM (Lag 3)	0,9047	0,8576	No ARCH Effect
Model Stability	Sign Bias Test (Joint)	0,3121	0,8629	No Asymmetry
Distributional Fit	Adjusted Pearson Goodness-of-Fit	0,7184	0,2758	Fit Distribution

According to Table 7, all test $p\text{-value} > 0,05$. This confirms that the residuals satisfy white noise conditions, are free from heteroskedasticity, show no asymmetric bias, and conform to the Student's t-distribution, thus validating their suitability for subsequent Copula modeling.

4.7 Standardized Residual Transformation via Probability Integral Transform (PIT)

The PIT transformation process was conducted by evaluating the standardized residuals through the cumulative distribution function (CDF) of the Student's t-distribution, utilizing the shape parameters estimated during the preceding GARCH stage. Validation of this transformation was examined through histogram analysis, Q-Q plots, and scatter plots

a. Verification of Uniform Distribution (Histogram)

The frequency distribution histograms of the PIT-transformed data are presented in Figure 9.

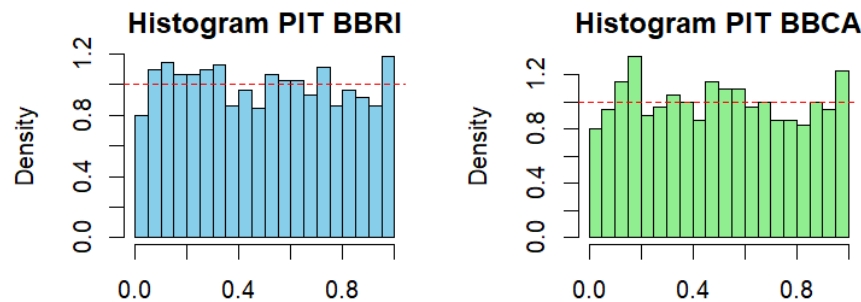


Figure 9. Frequency Histograms for BBRI and BBKA Stock Data

As shown in Figure 9, the frequency distributions exhibit a flat pattern relative to the theoretical density line. This confirms that the standardized log-return series of BBRI and BBKA have been successfully transformed into uniform distributions on the interval $[0,1]$.

b. Verification of Marginal Model Fit (Uniform Q-Q Plot)

A more detailed assessment was conducted using Q-Q plots to compare the sample empirical distribution against the theoretical uniform distribution, as illustrated in Figure 10.

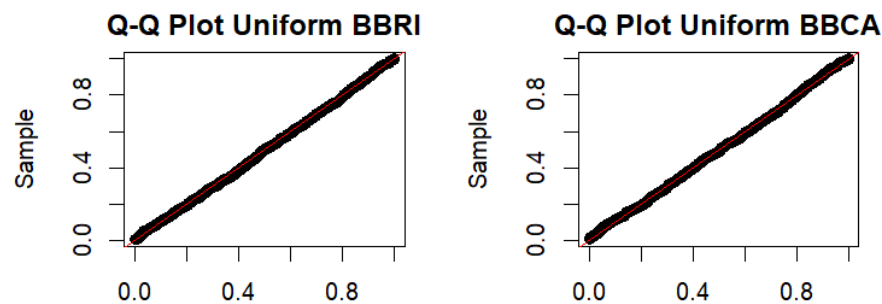


Figure 10. Uniform Q-Q Plots for BBRI and BBKA Stock Data

In Figure 10, the sample data points for both stocks align consistently along the diagonal line. This outcome demonstrates that the marginal GARCH(1,1) models with Student's t-distributed errors fit the empirical data structure exceptionally well.

c. Visualization of Dependency Structure (Scatter Plot)

The joint dependency structure between the two stocks within the uniform $[0,1]$ space is mapped via a scatter plot in Figure 11.

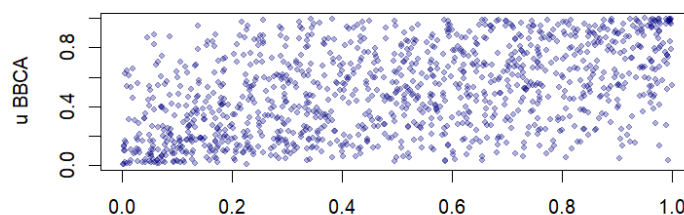


Figure 11. Scatter Plot of BBRI and BBKA Uniform Data

Figure 11 demonstrates that the data points are not randomly scattered, but rather form a positive linear pattern. The concentration of observations in the lower-left corner ($u_1 u_2 \approx 0$) and upper-right corner ($u_1 u_2 \approx 1$) indicates a strong dependency structure, particularly during co-occurring extreme price movements.

4.8 Non-Parametric Dependency Analysis (Kendall's Tau)

A preliminary analysis of non-parametric relationships was performed using the Kendall's Tau (τ) rank correlation coefficient on the PIT-transformed data. The calculation results are presented in Table 8.

Table 8. Kendall's Tau Correlation Matrix

Stock	BBRI	BBCA
BBRI	1,0000	0,3242
BBCA	0,3242	1,0000

The Kendall's Tau coefficient of 0,3242 indicates a moderate and statistically significant positive rank dependence. A higher return rank for BBRI stock consistently exhibits a concordant tendency to be accompanied by a higher return rank for BBCA stock.

4.9 Parameter Estimation Analysis of the t-Student Copula

Parametric estimation of the joint dependency structure was conducted using the Maximum Likelihood Estimation (MLE) method applied to the t-Student Copula model. The estimated parameters are summarized in Table 9.

Table 9. Parameter Estimation Results for the t-Student Copula

	Estimate	Std. Error
Rho (ρ)	0,4886	0,021
Degrees of Freedom (ν)	12,9364	5,556
Log-Likelihood	169,8	

Based on Table 9, the copula correlation parameter is estimated at $\rho = 0,4886$ with a small standard error (0,021), reflecting a significant positive linear association between the standardized residual series.

The estimated degrees-of-freedom parameter is $\nu = 12,9364$. This figure confirms the presence of symmetric tail dependence, indicating that BBRI and BBCA stock possess a measurable probability of co-occurring extreme movements during market crises. The maximized log-likelihood value of 169,80 demonstrates that the t-Student Copula provides a high goodness-of-fit in modeling the joint dependency structure of these two banking assets.

4.10 Portfolio Simulation via Monte Carlo Method

The next phase involved conducting a Monte Carlo simulation across 10.000 future return scenarios by integrating the t-Student Copula dependency structure along with conditional mean and volatility forecasts from the GARCH models. The portfolio was assumed to be equally weighted (50% allocated to BBRI stock and 50% to BBCA stock). A statistical summary of the simulated portfolio returns is presented in Table 10. Based on Table 10, the expected mean daily portfolio return is 0,00047 (0,047%) indicating slight positive growth under normal market conditions.

However, the primary focus of risk management lies in potential extreme outcomes. The worst-case scenario (minimum return) reveals a potential daily loss of up to 8,97% (-0,0897) whereas the maximum gain reaches 7,09% (0,0709). This wide spread reflects substantial price volatility resulting from the combination of high individual asset volatility and tail dependence between the banking stocks. This simulated return distribution serves as the direct input for Value at Risk (VaR) calculations.

Table 10. Statistical Summary of Simulated Portfolio Returns

Descriptive Statistic	Nilai Return
Minimum	-0,08977
1st Quartile (<i>1st Qu.</i>)	-0,00707
Median	0,00052
Mean	0,00047
3rd Quartile (<i>3rd Qu.</i>)	0,00808
Maximum	0,07097

4.11 Value at Risk (VaR) and Expected Shortfall (ES) Analysis

The final step in the risk assessment involves calculating potential portfolio losses from the simulated return distribution generated by the Monte Carlo method. Two primary risk metrics were evaluated: Value at Risk (VaR), which establishes the maximum expected loss at a specified confidence level, and Expected Shortfall (ES), which measures the average magnitude of losses whenever the VaR threshold is breached (tail loss). The estimation results are summarized in Table 11.

Table 11. VaR and ES Risk Estimation Results

Risk Indicator	95% Confidence Level	99% Confidence Level
Value at Risk	1,9774%	3,1301%
Expected Shortfall	2,7571%	4,0775%

As shown in Table 11, the daily 95% VaR of 1,9774% indicates 95% confidence that daily portfolio losses will not exceed this threshold under normal market conditions. To account for more severe market stress, the VaR at the 99% confidence level increases to 3,1301%.

Expected Shortfall (ES) provides a more conservative risk measurement during extreme conditions beyond the VaR threshold. At the 95%, confidence level, the ES value of 2,7571% represents the expected average loss faced by investors if daily losses exceed the 95% VaR limit. Under severe market shocks (99% confidence level), the expected average loss reaches 4,0775% of total portfolio value.

4.12 Model Validation (Backtesting)

Backtesting was performed to validate the accuracy of the t-Student Copula VaR model on historical data by assessing the occurrence of violations via Kupiec's Proportion of Failure (POF) test. The test results are detailed in Table 12.

Table 12. Summary of VaR Backtesting Results

Evaluation Indicator	VaR 95%	VaR 99%
Total Observations	1.205	1.205
Number of Violations	76	17
Actual Violation Proportion	6,31%	1,41%
Target Violation Proportion	5,00%	1,00%
<i>P-value</i> (Kupiec Test)	0,0450	0,1771

According to Table 12, at the 95% confidence level, the Kupiec test yields a *p-value* of 0,0450 ($< 0,05$), indicating that the model is slightly aggressive (underestimates risk) for moderate daily market fluctuations.

However, evaluation at the 99% confidence level (the standard mandated by the Basel Accord) reveals robust model performance. Recording 17 violations, the Kupiec test produces a *p-value* 0,1771 ($> 0,05$). This result confirms that the empirical failure rate is not statistically significantly different from the target rate of 1%, validating the efficacy of the joint GARCH–t-Student Copula approach in modeling tail dependence and fat-tailed behavior during market crises or extreme shocks.

4.13 Discussion

The portfolio risk estimation for BBRI and BBKA stock is strongly influenced by the non-normal characteristics of the marginal data, which exhibit high kurtosis (5,56 and 5,60, respectively). In line with findings by Ayala-Urrea & Hoyos-Giraldo (2022) regarding the fat-tailed phenomenon, the implementation of a GARCH(1,1) model with Student's t-distributed errors is crucial to address volatility clustering until the standardized residuals satisfy white noise assumptions.

Through t-Student Copula estimation, a significant positive correlation was identified ($\rho = 0,4886$) alongside a low degrees-of-freedom parameter ($\nu = 12,9364$). This finite value of ν confirms the existence of a tail dependence structure, indicating that both major banking assets exhibit a strong propensity for simultaneous severe downturns during periods of market distress (bearish conditions). These findings support Carvalho & Sáfadi (2020), who demonstrated that the t-Student Copula is markedly superior to the Gaussian Copula in capturing systemic risk and the probability of joint extreme events in distributional tails.

Furthermore, the Monte Carlo simulation reveals an increase in VaR estimates from 1,9774% at the 95% confidence level to 3,1301% at the 99% confidence level. The risk evaluation is further refined by the Expected Shortfall (ES) metrics, which reach 4,0775% at the 99% confidence level. Consistent with the arguments of González-Sánchez & Nave Pineda (2023), the ES indicator proves to be a coherent risk measure that provides a more conservative estimate of actual average losses when market crises exceed the VaR threshold.

The reliability of this model is successfully validated by Kupiec's backtesting test, which demonstrates high accuracy at the 99% confidence level with a *p-value* of 0,1771. Overall, the integrated GARCH–t-Student Copula framework proves robust and dependable in forecasting extreme portfolio risk for banking equities in alignment with Basel Accord standards.

5. Conclusion

Based on the research findings and discussion, the following conclusions are drawn:

1. The GARCH(1,1) t-Student Copula modeling framework is valid and accurate in integrating volatility dynamics and symmetric tail dependence between BBRI and BBKA stock. A 10.000 iteration Monte Carlo simulation for an equally weighted portfolio (50:50) yields daily portfolio Value at Risk (VaR) estimates of 1,9774% at the 95% confidence level and 3,1301% at the 99% confidence level. The reliability of this framework is corroborated by Kupiec's backtesting results, producing *p-value* of 0,0450 for 95% VaR and 0,1771 for 99% VaR.
2. Individually, BBRI stock displays higher volatility and greater standalone risk compared to BBKA stock. Individual VaR values for BBRI reach 2,49% at the 95% level and 4,05% at the 99% level, whereas BBKA exhibits lower individual risk at 1,89% (95% level) and 2,93% (99% level). This divergence in risk profiles empirically provides an effective diversification effect, successfully reducing the combined portfolio risk below the standalone risk level of BBRI stock.

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